SYNTHESIS OF SIGNALS FOR MULTI-FREQUENCY RADAR WITH PARAMETERS DETERMINED BY REAL INTERFERENCE SITUATION

The synthesis of the probing signal has been carried out, which makes it possible to increase the range of the radar in a real jamming environment. The article assumes that during location a lot of waves re-radiated by inhomogeneities located within the resolving volume of the radar arrive at the receiving point. The length of the paths passable by the waves is different and randomly changes in time so the waves come at the receiving point with a different and randomly changing relative delay time. Optimization of the probing signal parameters of a multi-frequency radar in the presented case has been carried out. The possibility of reducing modulating noise when using multi-frequency signals and rational choice of their parameters is shown. A multi-frequency signal with a rectangular envelope and parameters that provide an increase in the radar detection range is synthesized.

**Keywords:** multi-frequency radar; interference environment; probing signal synthesis.

**Introduction**

**Formulation of the problem.** A raid by enemy air attack weapons at low and extremely low altitudes may lead to the need to search for new methods to increase the range of their detection, and the presence of modulating and passive interference when locating targets may lead to the need to find methods and means of combating them.

The presented situation is characterized by the non-simultaneity of wave arrival and fluctuations of their group delay time, which causes signal distortion and limits the possible frequency band and duration of coherent processing of probing signals.

Signal distortions are all the more noticeable, the greater the delay time between individual waves, which decreases with narrowing of the transmitting and receiving antenna diagrams. Distortions can manifest themselves as modulating noise.

One of the ways to solve this problem in such a jamming environment can be the optimization of the probing signal parameters of a multi-frequency radar. The use of multi-frequency signals and the rational choice of their parameters can reduce the amount of modulating noise.

**Analysis of recent research and publications.** It was shown in [1–3] that regardless of the objects nature of synthesis and specific conditions, the problem is reduced to minimizing the distance between some sets in the corresponding space. It should be emphasized that such an approach to synthesis covers only deterministic problems.

Signal synthesis, like other optimization issues, is reduced to variation problems. It was shown in [2–3] that the synthesis problem is very closely related to the approximation problem.

The criteria for assessing the quality of approximation can be different, but the most common are quadratic and uniform (minimax). In the first case, they seek to minimize the quadratic difference of functions on a given interval \((-T/2, +T/2)\):

\[
\frac{T}{2} \int_{-T/2}^{T/2} |y(t) - x(t)|^2 \, dt = \min ,
\]

and in the second, the largest deviation of functions in the same interval –

\[
\max_{t \in T} |y(t) - x(t)| = \min .
\]

In the general case, the approximation criterion is determined by a condition of the type:

\[
\varepsilon(x, y) = \min ,
\]

\(\varepsilon – \) is positive functional, and minimization is performed over all possible \(x\).

The choice of an approximation criterion is almost always a difficult and controversial issue. There are often only intuitive considerations are used in such a choice, or preference is given to the criterion that leads more easily to a decision. As a rule, the criterion (1) is applied.

**The purpose of the article** consists in the development of a mathematical apparatus for the synthesis of signals for multi-frequency radars with parameters determined by the real jamming environment.

**Main material**

The problem of synthesizing a multi-frequency probing signal with a rectangular envelope, which has good correlation properties, can be solved using the method described in [5–8]. Its essence is as follows.

It is known that the autocorrelation function \(R(t)\)
uniquely determines the signal power spectrum:

$$|a(2\pi f)|^2 = \int_{-\infty}^{\infty} R(t)e^{-j2\pi ft} \, dt,$$

so all desired signals $x(t)$ multiplicity $X$ have the same amplitude spectrum $a(2\pi f)$, depending on the given $R(t)$:

$$x(2\pi f) = a(2\pi f)e^{-j\alpha(2\pi f)}. \quad (3)$$

Phase spectrum $a(2\pi f)$ is arbitrary, this is what distinguishes one signal of the set $X$ from another. Let there be an arbitrary admissible signal $y(t)$ with spectrum:

$$y(2\pi f) = b(2\pi f)e^{-j\beta(2\pi f)} \quad (4)$$

The proof is as follows:

a) the best approximation to the signal $y(t)$ with the spectrum (4) gives on the set $X$ signal $x(t)$, the spectrum of which is determined by the condition:

$$x(2\pi f) = a(2\pi f)e^{-j\beta(2\pi f)} \quad (5)$$

for all values $f$, at which $b(2\pi f) \neq 0$;

b) if the amplitude spectrum $b(2\pi f)$ is different from zero in every interval $2\pi f$ of finite measure, the signal of the best approximation on the set $X$ the only one;

c) minimum distance $d^2(y, X)$ between signals $y(t)$ and multitude $X$ and the corresponding proximity coefficient $C(y, X)$ with the quadratic criterion (1) equal:

$$d^2(y, X) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \left[|a(2\pi f)|^2 - |b(2\pi f)|^2\right] \, df,$$

$$C(y, X) = \sqrt{\int_{-\infty}^{\infty} |a(2\pi f)|^4 \, df \int_{-\infty}^{\infty} |b(2\pi f)|^4 \, df} \leq 1. \quad (7)$$

To get the shortest distance $d_{\text{min}}$, it is necessary to minimize the right part (6) by signals $y(t)$:

$$d^2_{\text{min}}(y, X) = \min_{y \in Y} \int_{-\infty}^{\infty} \left[|a(2\pi f)|^2 - |b(2\pi f)|^2\right] \, df,$$

or that is equivalent,

$$C(Y, X) = \max_{y \in Y} \sqrt{\int_{-\infty}^{\infty} |a(2\pi f)|^4 \, df \int_{-\infty}^{\infty} |b(2\pi f)|^4 \, df}. \quad (9)$$

Thus, the optimal permissible signal $y \in Y$, implementing distance $d_{\text{min}}$, gives the best quadratic approximation of the amplitude spectrum $b(2\pi f)$ to the given amplitude spectrum $a(2\pi f)$.

Approximation of amplitude spectra, achieved when applying the proximity criterion, ensures a certain approximation of the correlation function of the signal to the given one. In particular, there is an identity:

$$\int_{-\infty}^{\infty} |R(t) - R_y(t)|^2 \, dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} \left[a^2(2\pi f) - b^2(2\pi f)\right]^2 \, df,$$

showing that the best quadratic approximation of the correlation functions is achieved with a similar approximation of the power spectra – the squares of the amplitude spectra.

Suppose that the desired signal has a rectangular envelope that belongs to a set of multi-frequency signals, formed by the method of phase modulation and has the form of an amplitude-frequency spectrum corresponding to any weight window. In accordance with the assumptions, a signal with harmonic phase modulation can be taken as admissible (4). The expression for the complex envelope of such a signal has the form:

$$U(t) = U_m e^{jM_\phi \sin(2\pi f_n t + \phi_n)} = U_m \sum_{n=0}^{\infty} J_n(M_\phi) e^{j(2\pi f_n t + \phi_n)} \quad (10)$$

$U_m$ – is complex envelope amplitude; $M_\phi$ – phase modulation index; $f_n$ – modulation frequency of the harmonic modulating function; $\phi_n$ – initial phase of the harmonic modulating function (further $\phi_n = 0$).

$J_n(M_\phi)$ – Bessel functions of the first kind $\xi$ orders with modulation index $M_\phi$ as an argument.

An analysis of the expressions for the spectral density shows that they are real, which means that the phase frequency spectrum multi-frequency signal takes the values $0$ or $\pi$ [6–9].

The complex envelope of multi-frequency signal can be written:

$$y(t) = B(t)e^{j\varphi(t)}, \quad (11)$$

there for a rectangular envelope we have

$$B(t) = \begin{cases} \sqrt{\frac{c}{n}} & npu - \frac{\pi c}{2} < t < \frac{\pi c}{2}, \\ 0 & npu |t| > \frac{\pi c}{2}, \end{cases}$$

provided that the signal energy $E_s = 1$; $\varphi(t) = \beta \sin(2\pi f_n t)$ – phase modulation law.
The signal spectrum (11) is described by expression (4). According to (5), the desired signal $x(t)$ must have a phase frequency spectrum that coincides with the phase frequency spectrum of the signal with harmonic phase modulation. Then, in accordance with (3) we have:

$$x(t; \beta, \pi) = a(2\pi f) e^{-j\beta(2\pi f)}.$$

$\beta(2\pi f)$ is determined from the expression:

$$\beta(2\pi f) = b(2\pi f) = \int_{-\infty}^{\infty} B(t) e^{-j(\phi(t) - 2\pi f)} dt.$$

With the assumptions made, the proximity coefficient $C(x, X)$ depends only on the amplitude spectrum $b(2\pi f)$ and is given by the formula (7). To get the shortest distance $d_{\text{min}}$ it is necessary to maximize the coefficient of proximity by signals $y(t)$.

As a result, it is necessary to solve a variational problem. Required to define a function $\varphi(t) = \Phi(t)$, giving the maximum value:

$$C(x, X) = \frac{\int_{-\infty}^{\infty} a(2\pi f) b(2\pi f) df}{\sqrt{\int_{-\infty}^{\infty} [a(2\pi f)]^2 df \int_{-\infty}^{\infty} [b(2\pi f)]^2 df}} = \max,$$

$$b(2\pi f) = \int_{-\infty}^{\infty} B(t) e^{-j\phi(t) - 2\pi f} dt.$$ (14)

This expression determines the relationship between the shape of the spectrum of the harmonically modulated multi-frequency signal and the modulating function.

In accordance with the assumption that the phase-frequency spectra of the desired and permissible signals (5) are equal, it can be assumed that a similar dependence (15) also exists for the synthesized signal with phase modulation of a periodic voltage of a more complex shape, where an arbitrary phase-modulated periodic voltage oscillation is written in the form:

$$u(t) = U_m e^{j[2\pi f t + \phi(t)]} = U_m \sum_{\xi = -\infty}^{\infty} A_{\xi} e^{j[2\pi f t + \xi 2\pi f u]}.$$ (16)

Function $\varphi(t) = \Phi(t)$, that satisfies these conditions is the sought-for law of phase modulation. The solution of the mentioned variational problem encounters certain difficulties and does not always lead to a positive result.

It is possible to avoid solving the variational problem if we carry out a series of simple arguments.

The shape of the amplitude-frequency spectrum of a signal with harmonic phase modulation is determined by the dependence of the values of the Bessel functions $J_{\xi}(M_\phi)$ on the order of the functions $\xi $ (10). Frequency component amplitudes of $\xi$ order of the phase-shift keyed waveform are equal to the amplitude of the unmodulated waveform multiplied by the absolute value of the quantity $J_{\xi}(\beta)$.

Expression (10) for the complex envelope of a multi-frequency signal with harmonic phase modulation can be represented as:

$$U(t) = U_m \left[ J_{0}(M_\phi) + \sum_{\xi = 1}^{\infty} J_{\xi}(M_\phi) e^{j\xi 2\pi f u} \right] + \sum_{\xi = 1}^{2}(\xi 2\pi f u - \xi \pi / 2) \sin(\xi \pi / 2) + j \cos(\xi 2\pi f u - \xi \pi / 2) \sin(\xi \pi / 2)$$ (17)

After some simple transformations, we can get:

$$U(t) = U_m \left[ J_{0}(M_\phi) + 2 \sum_{\xi = 1}^{\infty} J_{\xi}(M_\phi) \cos(\xi 2\pi f u - \xi \pi / 2) \cos(\xi \pi / 2) \right],$$

Then the law of phase modulation is given by the expression:

$$\varphi(t) = \arctg \frac{2U_m \sum_{\xi = 1}^{\infty} J_{\xi}(M_\phi) \cos(\xi 2\pi f u - \xi \pi / 2) \sin(\xi \pi / 2)}{U_m J_{0}(M_\phi) + 2U_m \sum_{\xi = 1}^{\infty} J_{\xi}(M_\phi) \cos(\xi 2\pi f u - \xi \pi / 2) \cos(\xi \pi / 2)}.$$ (15)

It is noteworthy that in expression (16) there are no phase increments of the frequency components, which corresponds to condition (5).

In this case, given the distribution $A_{\xi}$ frequency components of the amplitude-frequency spectrum of the synthesized signal, it is possible to obtain the phase modulation law, under which condition (13) will be satisfied. Therefore, the expression for the required phase modulation law has the form:

$$\Phi(t) = \arctg \frac{2 \sum_{\xi = 1}^{\infty} A_{\xi} \cos(\xi 2\pi f u - \xi \pi / 2) \sin(\xi \pi / 2)}{A_0 + \sum_{\xi = 1}^{\infty} A_{\xi} \cos(\xi 2\pi f u - \xi \pi / 2) \cos(\xi \pi / 2)}.$$ (17)

The signal energy does not depend on the type of
modulating function with phase modulation, but depends on the energy of the modulated radio pulse, so you need to take into account that \( \sum_{n=-\infty}^{\infty} A_n^2 = 1 \).

It should also be noted that the amplitude-frequency spectrum of the signal is considered in a limited frequency band. Therefore, the limits of the sum in expressions (15-17) will be finite. The width of the amplitude-frequency spectrum of a signal with harmonic phase modulation can be estimated by the formula:

\[
\Delta f = (1 + M_\Phi + \sqrt{M_\phi}) f_\text{inter},
\]

(18)

\( M_\Phi \) – is modulation index.

This does not take into account the frequency components, the amplitude of which does not exceed 1% of the amplitude of the unmodulated carrier. The required width of the amplitude-frequency spectrum and the arrangement of frequency components are set initially for the synthesized signal. These values predetermine the number of components in the spectrum of a multifrequency signal.

Modulating voltages were synthesized \( \Phi(t) \) in accordance with expression (17), realizing multifrequency signals with an amplitude-frequency spectrum, the shape of which is close to the modulation functions.

Amplitudes of the frequency components of the spectrum \( \xi \) orders are the product of the tabulated values of the Bessel functions not only of different orders, but also of different modulation indices. This feature, as well as a finite number of frequency components, can explain the fact that it is impossible to synthesize a multi-frequency signal with an arbitrary shape of the amplitude-frequency spectrum with absolute accuracy.

In calculations, the duration of the signals was taken equal to 10 modulation periods \( \tau_c = 10T_\text{c} \). The width of the main lobe of the autocorrelation function is calculated relative to half the width of the main lobe of the autocorrelation function. Amplitude of the first sidelobe \( R_1 \) calculated as \( 20\log \frac{R_1}{R(0)} \) (there \( R(0) \) – is amplitude of the main peak of the autocorrelation function). In addition, the proximity factor \( C(y, X) \) (13), characterizing the “distance” between the desired signal and the synthesized one.

**Conclusions**

Calculations show that for any weight function and a different number of frequency components of the amplitude-frequency spectrum of the synthesized signal, the proximity coefficient \( C(y, X) \) always is more than 0.9. This indicates a high correlation of the spectra and the autocorrelation function of the ideal and synthesized signals, as well as the validity of all the assumptions made when solving the synthesis problem.

Unfortunately, there is the fact that, despite the high value of the proximity coefficient \( C(y, X) \), an amplitude of the first side lobe of the autocorrelation function of the synthesized signals is somewhat larger than in the case of ideal and desired signals. Such a difference in the levels of the side lobe of the autocorrelation function of ideal and synthesized signals is due to the fact that the amplitudes of the frequency components of the spectrum are distributed in a complex way over combinations of Bessel functions. The latter, in their turn, have strictly tabulated values. Therefore, it is impossible to realize any form of the amplitude-frequency spectrum with absolute accuracy. The obtained results prove that the value \( C(y, X) \) is an integral criterion.

As a result, the synthesis of signals for multifrequency radars is considered, which allows to obtain a modulating voltage that best implements such a signal for a given number of frequency components and the shape of the amplitude-frequency spectrum of a multifrequency signal.

Thus, the problem of signal synthesis for multifrequency radars with parameters determined by the real interference situation has been solved.

**References**

1. Николаев А. И., Ширман Я. Д. О возможности приближения к широкополосным шумоподобным сигналам при сохранении одноканальности (малоканальности) обработки. Вопросы обработки радиолокационных сигналов и помех. РЛС. 1973. С. 11–17.
Проведено синтез зондувального сигналу, який дозволяє збільшувати дальність дії РЛС у реальній перешкодовій обстанові. У статті пропонується, що при локації в точку прийому прикладає безліч перевищень неодночасності електромагнітних хвилин (ЕМХ). Довжина хвиль, що проходять ЕМХ, різка і виходячо змінюється в часі, тому ЕМХ приходять в точку прийому з різним відносним часом запізнення, що виходячо змінюється. Неодночасність приходу ЕМХ та флукутації їх групового часу запізнення викликає спотворення сигналу та обмежує можливу частоту та тривалість когерентної обробки зондувальних сигналів (смугу частот і тривалості когерентності). Спостереження сигналу тим більше помітні, чим більше час запізнення між окремими ЕМХ, який зменшується при звуженні діапазону спектру перешкоди та прийомників антен. Спостереження перешкоди обчислюють за формулою, що приймає урахування впливу на перешкоди багаточастотної РЛС. Проведено оптимізацію параметрів зондувального сигналу багаточастотної РЛС у такій перешкодовій обстановці. Показала можливість змінення впливу перешкоди, при корекції багаточастотних сигналів і різноманітному виборі їх параметрів. Розрахунки показують, що для будь-якої вагової функції та різної кількості частотних складових амплітудно-частотного спектру синтезованого сигналу коекфіцієнт відносної кількості змін перешкоди менші, ніж 0,9. Це свідчить про високу кореляцію спектрів та автоматизаційної функції ідеального та синтезованого сигналу, а також правомірність їх прийняття, допущених при вирішенні задачі синтезу. Синтезований багаточастотний сигнал з прямокутним огінником та параметрами, які забезпечують відповідність діаграм відображення РЛС. Розглянуто синтез сигналів для багаточастотних РЛС, що дозволяє для заданого числа частотних складових і форми амплітудно-частотного спектра багаточастотного сигналу отримати модулюючу напругу, що найкраще реалізує такий сигнал. Таким чином, вирішення задачі синтезу сигналів для багаточастотних РЛС з параметрами, що визначаються реальним перешкодовим обстановою.